CHAPTER 25 Capacitors and Dielectrics

Answers to Understanding the Concepts Questions

1. The fact that there are two ways of writing the units of permittivity changes nothing. The flexibility may allow for ease in the cancellation of units in different equations, but this is purely a matter of convenience.

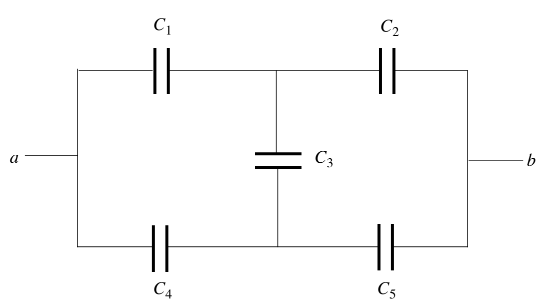
2. Form a parallel-plate capacitor with the two parallel metal plates. Charge the capacitor by hooking it up to the battery, and measure the potential difference V across the capacitor with the voltmeter. The charge on the capacitor must satisfy $Q = CV$. Next, disconnect the battery from the capacitor, so that Q cannot change anymore. Now insert the plastic to completely fill the air gap in between the two metal plate. As a result the capacitance of the capacitor becomes $C' = \kappa C$, with κ the dielectric constant of the plastic. Since Q cannot change, the voltage difference V', which again can be measured by the voltmeter, must satisfy $Q = C'V' = \kappa CV'$. Thus $CV = \kappa CV'$, and $\kappa = V/V'$.

3. The reminder provides the proof: Consider a closed path which consists of a segment between the plates, leading from one to the other, and a segment which closes the path outside of the plates. If there is a voltage drop in the interior region, there must be a voltage rise outside that region. This would be impossible if the electric field vanished outside.

4. By definition $C = Q/V$. If $C = 0$, then $Q = 0$, meaning that no charge can be stored on the capacitor without introducing an infinite V. This can happen, for example, as the cross-sectional area of a parallel-plate capacitor approaches zero.

5. The reason the capacitance per unit length goes to zero is that in the limit under consideration, the potential difference becomes infinitely large. It takes infinite work to concentrate an infinite amount of charge along an infinitely long line.

6. An example is shown below.



7. A capacitor consists of two separate metal pieces that are insulated from one another. When a voltage difference is applied across the two metal pieces, one piece is charged to + Q while the other is charged to – Q. The net charge on the entire capacitor, including both metal pieces, is therefore always zero.

8. The fact that unlike charges attract and like charges repel forces the dielectric constant to be larger than or equal to unity.

9. Since the plates are disconnected from the battery Q cannot change, and neither can the electric field E in between the plates (as it is proportional to the surface charge density on each plate). If the two plates are pushed closer together, the potential difference between them, $V = Ed$, must therefore decrease along with d, the plate separation). Thus $C = Q/V$ must increase. (This is also clear from the expression for the capacitance of the parallel-plate capacitor). Meanwhile, the electrostatic energy store in the capacitor, $Q^2/2C$, must decrease as Q is fixed whiled C increases. This is also understood from the fact that, while the energy density in between the two plates remains the same (as it is proportional to E^2, which does not change), the volume of the region in between the two plates decreases.

10. When V is held fixed, Q is proportional to C. When the plates are pushed together, C increases, and so must Q. Another way to see this is to note that if the potential difference between the plates is fixed as the distance between the plates decreases, then the electric field must grow. This can only happen if the surface charge density, and thus the total charge, grows. What about energy? For fixed V, the energy is proportional to C (or Q). Thus pushing the plates together increases the energy, and positive work must be done to push the plates together. Alternatively, note that the energy is proportional to E^2 times the interior volume. The volume decreases as the distance between the plates decreases, but the electric field grows in the same way. Thus E^2 grows quadratically with the separation, and the total energy is proportional to the separation.

11. The capacitance of a spherical capacitor is proportional to its radius, which is fixed if the surface area A is fixed. There is not much we can do to adjust its capacitance. If we form a parallel-plate capacitor or a concentric-cylinder capacitor, then theoretically C can be made arbitrarily large by making the separation between the two metal pieces arbitrarily small. Note, however, that as the separation between the two concentric cylinders become very small, the capacitance of the concentric-cylinder arrangement approaches that of the parallel-plate capacitor (for the same plate area). So either of them can be used to make a capacitor of large capacitance by reducing the plate separation.

12. The two plates of a large charged capacitor carry charges Q and $-Q$; these charges may be large. When a wire connects the plates the charge will flow through the wire, generally in a very short time. This could be dangerous if the person making the connection is careless and allows some of the charge to flow through him or her. More generally, a large amount of energy has been stored in the capacitor and is dissipated when the charges flow out. It is always a potential danger when a large amount of energy is dissipated over a short time interval.

13. Consider a pair of plates carrying charges Q and $-Q$, respectively. Without the fringe effect, the electric filed would abruptly reduce to zero at the edge of the plates. The fringe effect causes the electric field to "leak" outside. Since the total electric flux depends on the charge on each plate and cannot change, the electric field inside must be somewhat diluted as a result. Therefore, for the same Q, the potential difference V between the two plates is weaker due to fringe effect, and the capacitance, $C = Q/V$, increases as a result.

14. Two oppositely charged nonconductors will give rise to an electric field between them, with the field lines going from the positive to the negative charges. This configuration will store electrical energy and act in that sense just like a capacitor. The difficulty is that it is hard to put a large charge on insulators, and it is also hard to discharge them. That is why conductors are much more useful.

15. According to the calculation in Example 25-5, the battery contains some 600 million times the energy it takes to charge up a single capacitor. So it can certainly be used to charge more than one of them. In fact, it would only take 1/600,000 of the energy of the battery to charge 1000 such capacitors.

16. If the plates are not shorted, they could accumulate charges and produce a voltage across them, and that can be hazardous should your candle such a capacitor improperly and it gets discharged by driving a current through your body.

17. The question is whether the vacuum can be polarized; that is, whether positive and negative charges can be separated from it. In terms of what we know, there are no charges in the vacuum, and therefore an electric field applied to it will not induce a charge separation. When you study quantum mechanics you will learn that the full answer to this question is quite different from the answer given here.

18. The air-filled capacitors are usually operated at a high frequency, i.e., they are hooked up to a rapidly alternating voltage source that quickly charges and discharges the capacitor, so the slow leakage of charges through the air gap does not present a significant problem.

Solutions to Problems

1. (a) For a parallel-plate capacitor, we have

$$C = \varepsilon_0 A/d$$
$$= (8.85 \times 10^{-12}\ \text{F/m})(50 \times 10^{-4}\text{m}^2)/(1 \times 10^{-3}\ \text{m}) = 4.4 \times 10^{-11}\ \text{F} = \boxed{44\ \text{pF}}.$$

(b) For a sphere, we have

$$C = 4\pi\varepsilon_0 R;$$
4.4×10^{-11} F $= 4\pi(8.85 \times 10^{-12}$ F/m$)R$, which gives $R = 0.40$ m $= \boxed{40\ \text{cm}}$.

2. For a coaxial cable, we have

$$C = L2\pi\varepsilon_0/\ln(R_2/R_1)$$
$$= (1.0 \times 10^3\ \text{m})2\pi(8.85 \times 10^{-12}\ \text{F/m})/\ln[(1.2\ \text{cm})/(0.8\ \text{cm})] = 1.4 \times 10^{-7}\ \text{F} = \boxed{0.14\ \mu\text{F}}.$$

3. From $V = Q/C$, we have

(a) $V = (4\ \mu\text{C})/(4\ \mu\text{F}) = \boxed{1\ \text{V}}$.

(b) $V = (10\ \mu\text{C})/(4\ \mu\text{F}) = \boxed{2.5\ \text{V}}$.

(c) $V = (1 \times 10^{-3}\ \text{C})/(4 \times 10^{-6}\ \text{F}) = \boxed{250\ \text{V}}$.

4. From $Q = CV$, we have

(a) $Q = (1\ \mu\text{F})(2\ \text{V}) = \boxed{2\ \mu\text{C}}$.

(b) $Q = (1\ \mu\text{F})(12\ \text{V}) = \boxed{12\ \mu\text{C}}$.

5. For a parallel-plate capacitor, we have

$$C = Q/V = \varepsilon_0 A/d;$$
$(2 \times 10^{-6}\ \text{C})/(3000\ \text{V}) = (8.85 \times 10^{-12}\ \text{F/m})(2.5 \times 10^{-2}\text{m}^2)/d$, which gives $d = 3.3 \times 10^{-4}\ \text{m} = \boxed{0.33\ \text{mm}}$.

Because V is the maximum of the power supply, this is the maximum separation. Note that

$$E = V/d = (3000\ \text{V})/(3.3 \times 10^{-4}\ \text{m}) = 9 \times 10^6\ \text{V/m},$$
which is greater than the dielectric strength of air; the plates must be evacuated.

6. For a coaxial cable, we have

$$C = L2\pi\varepsilon_0/\ln(R_2/R_1)$$
$$= (1.8\ \text{m})2\pi(8.85 \times 10^{-12}\ \text{F/m})/\ln[(1.5\ \text{cm})/(1.0\ \text{cm})] = 2.5 \times 10^{-10}\ \text{F} = \boxed{0.25\ \text{nF}}.$$

7. Because the potential from the outer conductor is constant inside, the potential difference between the two conductors is due to the inner conductor, which is equivalent to a point charge:

$V = (Q/4\pi\varepsilon_0)[(1/r) - (1/R)]$, so the capacitance is

$$C = Q/V = 4\pi\varepsilon_0 rR/(R - r).$$

(a) When r is finite and $R \to \infty$, we have

$C \to \boxed{4\pi\varepsilon_0 r}$, which is the capacitance of the inner sphere.

(b) When $(R - r) \ll r$, we have $R \to r$, so

$C \to 4\pi\varepsilon_0 r^2/(R - r) = \boxed{\varepsilon_0 A/d}$, which is the capacitance of parallel plates with separation d.

8. From Problem 7, we have

$C = 4\pi\varepsilon_0 rR/(R - r)$ and $V = Q/C = Q(R - r)/4\pi\varepsilon_0 rR$;

$$V = (1.4 \times 10^{-7}\ \text{C})(15 \times 10^{-2}\ \text{m} - 3.0 \times 10^{-2}\ \text{m})/[4\pi(8.85 \times 10^{-12}\ \text{F/m})(3.0 \times 10^{-2}\ \text{m})(15 \times 10^{-2}\ \text{m})]$$
$$= 3.4 \times 10^4\ \text{V} = \boxed{34\ \text{kV}}.$$

$\boxed{9}$. For a parallel-plate capacitor, we have

$$C = q/V = \varepsilon_0 A/d;$$

$$d = \varepsilon_0 AV/q = (8.85 \times 10^{-12}\ \mathrm{F/m})(4.0 \times 10^{-2}\,\mathrm{m}^2)[50.0\ \mathrm{mV} + (0.10\ \mathrm{mV/s})t](10^{-3}\ \mathrm{V/mV})/(4.0 \times 10^{-8}\ \mathrm{C})$$

$$= \boxed{(4.43 \times 10^{-7}\ \mathrm{m}) + (8.85 \times 10^{-10}\ \mathrm{m/s})t}.$$

10. For a parallel-plate capacitor, we have

$$E = \sigma/\varepsilon_0 = V/d;$$

$$\sigma = \varepsilon_0 V/d$$

$$= (8.85 \times 10^{-12}\ \mathrm{F/m})(3\mathrm{V})/(0.3 \times 10^{-3}\ \mathrm{m}) = \boxed{8.9 \times 10^{-8}\,\mathrm{C/m^2}}.$$

The total charge on each plate is

$$Q = \sigma A = (8.9 \times 10^{-8}\,\mathrm{C/m^2})(0.06\ \mathrm{m})^2 = 3.2 \times 10^{-10}\,\mathrm{C} = \boxed{0.3\ \mathrm{nC}}.$$

11. When the capacitor is isolated, the charge must be constant, so we have

$$Q = C_{\min}V_{\max} = C_{\max}V_{\min};$$

$$V_{\max} = (C_{\max}/C_{\min})V_{\min}$$

$$= [(0.2\ \mu\mathrm{F})/(0.01\ \mu\mathrm{F})](300\ \mathrm{V}) = 6 \times 10^3\ \mathrm{V} = \boxed{6\ \mathrm{kV}}.$$

12. (a) The capacitance of the system is

$$C = Q/V$$

$$= (900\ \mathrm{C})/(90 \times 10^6\ \mathrm{V}) = 10 \times 10^{-6}\ \mathrm{F} = \boxed{10\ \mu\mathrm{F}}.$$

(b) The energy stored in the system is

$$U = \tfrac{1}{2}QV$$

$$= \tfrac{1}{2}(900\ \mathrm{C})(90 \times 10^6\ \mathrm{V}) = \boxed{4.1 \times 10^{10}\,\mathrm{J}}.$$

13. The energy stored in the capacitor is

$$U = \tfrac{1}{2}QV = \tfrac{1}{2}(0.068\ \mathrm{C})(2900\ \mathrm{V}) = \boxed{99\ \mathrm{J}}.$$

14. The energy stored in the capacitor is

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(0.7\ \mathrm{pF})(2\ \mathrm{V})^2 = 0.98\ \mathrm{pJ} = \boxed{1\ \mathrm{pJ}}.$$

15. We find the capacitance from the energy stored in the capacitor:

$$U = \tfrac{1}{2}CV^2;$$

$27\ \mathrm{J} = \tfrac{1}{2}C(300\ \mathrm{V})^2$, which gives $C = 6.0 \times 10^{-4}\ \mathrm{F} = \boxed{600\ \mu\mathrm{F}}$.

16. For a sphere, we have $C = 4\pi\varepsilon_0 R$, so the energy stored is

$$U = \tfrac{1}{2}Q^2/C = \tfrac{1}{2}Q^2/4\pi\varepsilon_0 R$$

$$= \tfrac{1}{2}(3.0 \times 10^{-5}\ \mathrm{C})^2/4\pi(8.85 \times 10^{-12}\ \mathrm{F/m})(35 \times 10^{-2}\,\mathrm{m}) = \boxed{12\ \mathrm{J}}.$$

$\boxed{17}$. (a) For a coaxial cable, we have

$$C = L2\pi\varepsilon_0/\ln(b/a)$$

$$= (10\ \mathrm{m})2\pi(8.85 \times 10^{-12}\ \mathrm{F/m})/\ln[(8\ \mathrm{mm})/(3\ \mathrm{mm})] = \boxed{5.67 \times 10^{-10}\ \mathrm{F}}.$$

(b) The energy stored in 10 m of cable is

$$U_1 = \tfrac{1}{2}CV^2$$

$$= \tfrac{1}{2}(5.67 \times 10^{-10}\ \mathrm{F})(10^3\ \mathrm{V})^2 = \boxed{2.83 \times 10^{-4}\,\mathrm{J}}.$$

Because the capacitance is directly proportional to the length, the energy stored in 1 km of cable is

$$U_2 = [(10^3\ \mathrm{m})/(10\ \mathrm{m})]U_1 = \boxed{2.83 \times 10^{-2}\,\mathrm{J}}.$$

18. From Problem 7, we have $C = 4\pi\varepsilon_0 rR/(R - r)$ so the stored energy is

$$U = \tfrac{1}{2}Q^2/C = \tfrac{1}{2}Q^2(R - r)/4\pi\varepsilon_0 rR$$
$$= \tfrac{1}{2}(6.0 \times 10^{-8}\ \text{C})^2\ (12\ \text{cm} - 4\ \text{cm})(10^{-2}\ \text{m/cm})/[4\pi(8.85 \times 10^{-12}\ \text{F/m})(12\ \text{cm})(4\ \text{cm})(10^{-2}\ \text{m/cm})^2]$$
$$= 2.7\times 10^{-4}\ \text{J} = \boxed{0.27\ \text{mJ}}.$$

19. (a) When the plates are pulled apart, the charge does not change, while both the capacitance and the potential difference will change. The initial capacitance and charge are

$$C_0 = \varepsilon_0 A/d_0\ ;\quad Q_0 = C_0 V_0 = \varepsilon_0 A\, V_0/d_0 = Q.$$

If we express the energy stored in the capacitor as

$$U = \tfrac{1}{2}Q^2/C = \tfrac{1}{2}Q^2 d/\varepsilon_0 A,$$

the change in stored energy is

$$\Delta U = \boxed{\tfrac{1}{2}({Q_0}^2/\varepsilon_0 A)(d_1 - d_0) = (\varepsilon_0 A\, {V_0}^2/2{d_0}^2)(d_1 - d_0)}.$$

(b) Because the external force is the only interaction with the capacitor, we have

$$W_F = \Delta U = \boxed{(\varepsilon_0 A\, {V_0}^2/2{d_0}^2)(d_1 - d_0)}.$$

(c) If the plates stay connected to the battery, the potential difference does not change, while both the charge and capacitance will change. The change in stored energy is

$$\Delta U = \tfrac{1}{2}{V_0}^2(C - C_0) = (\varepsilon_0 A\, {V_0}^2/2)[(1/d_1) - (1/d_0)] = \boxed{(\varepsilon_0 A\, {V_0}^2/2d_1 d_0)(d_0 - d_1)}.$$

(d) Even though there must still be work done by the external force to separate the opposite charges on the plates, the energy stored in the capacitor decreases ($d_0 < d_1$). The charge on the plates has decreased and $\boxed{\text{energy has been stored in the battery}}$.

20. The energy stored in the electric field is

$$U_1 = \tfrac{1}{2}\varepsilon_0 E^2\ (\text{volume})_1$$
$$= \tfrac{1}{2}(8.85 \times 10^{-12}\ \text{F/m})(1.25 \times 10^5\ \text{V/m})^2(1\ \text{m}^3) = \boxed{6.91 \times 10^{-2}\ \text{J}}.$$
$$U_2 = \tfrac{1}{2}\varepsilon_0 E^2\ (\text{volume})_2$$
$$= \tfrac{1}{2}(8.85 \times 10^{-12}\ \text{F/m})(1.25 \times 10^5\ \text{V/m})^2(10^3\ \text{m})^3 = \boxed{6.91 \times 10^7\ \text{J}}.$$

21. The energy density around the long wire is

$$u = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0(\lambda/2\pi\varepsilon_0 r)^2 = \boxed{\lambda^2/8\pi^2\varepsilon_0 r^2}.$$

22. For a sphere, we have $C = 4\pi\varepsilon_0 R$. The energy stored in the electric field is the energy stored in the capacitor:

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(4\pi\varepsilon_0 R)V^2 = 2\pi\varepsilon_0 R\, V^2$$
$$= 2\pi(8.85 \times 10^{-12}\ \text{F/m})(0.75\ \text{m})(2.0 \times 10^4\ \text{V})^2 = 1.7\times 10^{-2}\ \text{J} = \boxed{17\ \text{mJ}.}$$

23. Because the cube is small compared to the distance from the point charge, we approximate the field in the cube from the point charge as constant:

$$E_{\text{av}} = (1/4\pi\varepsilon_0)(q/r^2) = (9 \times 10^9\ \text{m/F})[(5 \times 10^{-4}\ \text{C})/(1\ \text{m})^2] = 45 \times 10^5\ \text{V/m};$$
$$U = \tfrac{1}{2}\varepsilon_0 {E_{\text{av}}}^2 L^3 = \tfrac{1}{2}\varepsilon_0[(1/4\pi\varepsilon_0)(q/r^2)]^2 L^3 = (1/4\pi\varepsilon_0)q^2 L^3/8\pi r^4$$
$$= (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5 \times 10^{-4}\ \text{C})^2(5 \times 10^{-2}\ \text{m})^3/[8\pi(1\ \text{m})^4]$$
$$= \boxed{1.1 \times 10^{-2}\ \text{J}}.$$

24. For the energy density of the uniform field we have

$$u = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0(V/d)^2;$$

$10^{-6}\ \text{J/m}^3 = \tfrac{1}{2}(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)V^2/(1 \times 10^{-2}\ \text{m})^2$, which gives

$$V = \boxed{48\ \text{V}}.$$

$\boxed{25}$. For a conducting sphere, we have

$V = (1/4\pi\varepsilon_0)(q/r)$:

$8.3 \times 10^3\ \text{V} = (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)q/(18 \times 10^{-2}\ \text{m})$, which gives

$q = \boxed{1.7 \times 10^{-7}\ \text{C}}$.

The energy density outside the sphere is

$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0[(1/4\pi\varepsilon_0)(q/r^2)]^2$

$= \frac{1}{2}(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)[(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.7 \times 10^{-7}\ \text{C})/r^2]^2$

$= \boxed{1.0 \times 10^{-5}/r^4\ \text{J/m}^3\text{, with } r \text{ in m}}$.

Because there is no field inside the sphere, we find the total energy in the electric field by adding the energies in spherical shells with radius r and thickness dr:

$$U = \int_R^\infty \frac{1.0 \times 10^{-5}\ \text{J}\cdot\text{m}}{r^4} 4\pi r^2\,\mathrm{d}r = \left(4.0\pi \times 10^{-5}\ \text{J}\cdot\text{m}\right)\int_R^\infty \frac{\mathrm{d}r}{r^2} = \left(4.0\pi \times 10^{-5}\ \text{J}\cdot\text{m}\right)\left(-\frac{1}{r}\right)\Big|_R^\infty$$

$$= \left(4.0\pi \times 10^{-5}\ \text{J}\cdot\text{m}\right)\left(-\frac{1}{\infty} + \frac{1}{R}\right) = \left(4.0\pi \times 10^{-5}\ \text{J}\cdot\text{m}\right)\left(-0 + \frac{1}{R}\right) = \frac{4.0\pi \times 10^{-5}\ \text{J}\cdot\text{m}}{0.18\ \text{m}} = 7.0 \times 10^{-4}\ \text{J}.$$

Note that this is $\frac{1}{2}qV$.

26. Because the sphere is conducting, there is no field inside. The field outside is

$E_{\text{outside}} = Q/4\pi\varepsilon_0 r^2$.

Using the technique of Problem 25, we find the energy in the spherical region between $R = 25$ cm and $r = 50$ cm:

$U = \int(\varepsilon_0/2)(Q/4\pi\varepsilon_0 r^2)^2 4\pi r^2\,\mathrm{d}r$

$= (Q^2/8\pi\varepsilon_0)\int \mathrm{d}r/r^2$

$= (Q^2/8\pi\varepsilon_0)(-1/r)$

$= (Q^2/8\pi\varepsilon_0)(-1/r + 1/R)$

$= [(6.0 \times 10^{-7}\ \text{C})^2/8\pi(8.85 \times 10^{-12}\ \text{F/m})][-1/(0.50\ \text{m}) + 1/(0.25\ \text{m})]$

$= 3.2 \times 10^{-3}\ \text{J} = \boxed{3.2\ \text{mJ}}$.

27. (a) For a coaxial cable, we have

$C = L2\pi\varepsilon_0/\ln(b/a)$

$= (0.15\ \text{m})2\pi(8.85 \times 10^{-12}\ \text{F/m})/\ln[(2\ \text{cm}/0.02\ \text{cm})]$

$= 1.8 \times 10^{-12}\ \text{F} = \boxed{1.8\ \text{pF}}$.

(b) The energy that recharges the tube is stored in the capacitor:

$U = \frac{1}{2}CV^2 = \frac{1}{2}(1.8 \times 10^{-12}\ \text{F})(5 \times 10^2\ \text{V})^2 = \boxed{2.3 \times 10^{-7}\ \text{J}}$.

28. From Chapter 24, we have the electric field for a uniformly charged nonconducting sphere:

$E_{\text{inside}} = Qr/4\pi\varepsilon_0 R^3$ and $E_{\text{outside}} = Q/4\pi\varepsilon_0 r^2$.

Using the technique of Problem 25, we find the energy in the spherical region of radius $r = 0.25$ m

$U = \int(\varepsilon_0/2)(Qr/4\pi\varepsilon_0 R^3)^2 4\pi r^2\,\mathrm{d}r + \int(\varepsilon_0/2)(Q/4\pi\varepsilon_0 r^2)^2 4\pi r^2\,\mathrm{d}r$

$= (Q^2/8\pi\varepsilon_0 R^6)\int r^4\,\mathrm{d}r + (Q^2/8\pi\varepsilon_0)\int \mathrm{d}r/r^2$

$= (Q^2/8\pi\varepsilon_0 R^6)(r^3/5) + (Q^2/8\pi\varepsilon_0)(-1/r)$

$= (Q^2/8\pi\varepsilon_0)(1/5R - 1/r + 1/R)$

$= (Q^2/8\pi\varepsilon_0)(6/5R - 1/r)$

$= [(7.3 \times 10^{-6}\ \text{C})^2/8\pi(8.85 \times 10^{-12}\ \text{F/m})](1.2/0.07 - 1/0.25)$

$= \boxed{3.1\ \text{J}}$.

29. (a) We find the electric field between two parallel plates from

$E = V/d = (1500\text{ V})/(0.5 \times 10^{-2}\text{ m}) = \boxed{3.00 \times 10^5\text{ V/m}}$.

(b) In terms of the charge density on the plates, the field is $E = \sigma/\varepsilon_0$, which gives

$Q = \sigma A = \varepsilon_0 EA = (8.85 \times 10^{-12}\text{ F/m})(3.00 \times 10^5\text{ V/m})(400 \times 10^{-4}\text{m}^2) = \boxed{1.06 \times 10^{-7}\text{ C}}$.

(c) We have to remember that the field between the plates is produced by both plates, but the field that one plate produces at the other is one-half of this. The force exerted on one plate is

$F = \frac{1}{2}QE = \frac{1}{2}(1.06 \times 10^{-7}\text{ C})(3.00 \times 10^5\text{ V/m}) = \boxed{1.59 \times 10^{-2}\text{ N}}$.

(d) When the plates are pulled apart, the charge does not change, while both the capacitance and the potential difference will change. If we express the energy stored in the capacitor as

$U = \frac{1}{2}Q^2/C = \frac{1}{2}Q^2 d/\varepsilon_0 A$,

the change in stored energy is

$\Delta U = \frac{1}{2}(Q^2/\varepsilon_0 A)(d_2 - d_1)$

$= \frac{1}{2}[(1.06 \times 10^{-7}\text{ C})^2/(8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(400 \times 10^{-4}\text{m}^2)](0.20)(0.5 \times 10^{-2}\text{ m}) = 1.59 \times 10^{-5}\text{ J}.$

To pull the plates apart requires a force to balance the attractive force from part (c).
The work done by this force is

$W = F\,\Delta d = (1.59 \times 10^{-2}\text{ N})(0.2)(0.5 \times 10^{-2}\text{ m}) = \boxed{1.59 \times 10^{-5}\text{ J}}$, consistent with part (c).

30. (a) The electric field outside a charged sphere is that of a point charge:

$E = \boxed{e/4\pi\varepsilon_0 r^2,\ r > R}$.

(b) We can use the result of Problem 26, with $a \to \infty$, to find the energy stored in the electric field:

$U = e^2/8\pi\varepsilon_0 R$.

We could also find the energy from

$U = \frac{1}{2}eV = \frac{1}{2}e(e/4\pi\varepsilon_0 R) = e^2/8\pi\varepsilon_0 R$

$= (1.6 \times 10^{-19}\text{ C})^2/8\pi(8.85 \times 10^{-12}\text{ F/m})R = \boxed{(1.15 \times 10^{-28}\text{ J}\cdot\text{m})/R}$.

(c) If the energy stored in the electric field is the rest energy, we have

$e^2/8\pi\varepsilon_0 R = mc^2$;

$(1.6 \times 10^{-19}\text{ C})^2/8\pi(8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)R = (0.9 \times 10^{-30}\text{ kg})(3 \times 10^8\text{ m/s})^2$, which gives

$R = \boxed{1.4 \times 10^{-15}\text{m}}$.

31. For the two combinations we have

$C_p = C_1 + C_2$; $6.5\ \mu\text{F} = C_1 + C_2$, and

$1/C_s = 1/C_1 + 1/C_2$ or $C_s = C_1C_2/(C_1 + C_2) = C_1C_2/C_p$; $1.4\ \mu\text{F} = C_1C_2/(6.5\ \mu\text{F})$.

When we combine these two equations, we get a quadratic equation for C_1:

$C_1^2 - (6.5\ \mu\text{F})C_1 + 9.1\ \mu\text{F}^2 = 0$.

The two solutions are $C_1 = 2.04\ \mu$F and $4.46\ \mu$F. Thus the two capacitors are $\boxed{2.04\ \mu\text{F},\ \ 4.46\ \mu\text{F}}$.

32. The voltage must be the same across both the top and bottom sections, so we have

$E_1 = V/\ell_1 = \sigma_1/\varepsilon_0$,

which gives the charge on the top section: $Q_1 = \varepsilon_0(V/\ell_1)A_1$;

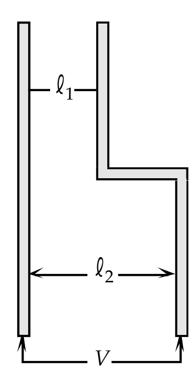
$E_2 = V/\ell_2 = \sigma_2/\varepsilon_0$,

which gives the charge on the bottom section: $Q_2 = \varepsilon_0(V/\ell_2)A_2$.

The areas are equal, $A_1 = A_2 = A/2$, and the total charge on the capacitor is

$Q = Q_1 + Q_2 = \varepsilon_0 V(A_1/\ell_1 + A_2/\ell_2)$

$= \varepsilon_0 V(A/2)(1/\ell_1 + 1/\ell_2) = \varepsilon_0 VA(\ell_1 + \ell_2)/2\ell_1\ell_2$.



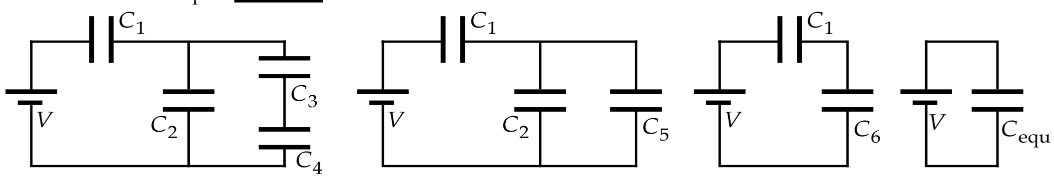
From the definition of capacitance, we have

$C = Q/V = \boxed{\varepsilon_0 A(\ell_1 + \ell_2)/2\ell_1\ell_2}$.

This is the equivalent capacitance for 2 capacitors in parallel:

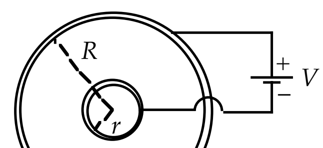
$C = (\varepsilon_0 A/2\ell_1) + (\varepsilon_0 A/2\ell_2) = C_1 + C_2$.

33. From the redrawn circuit, we see that C_3 and C_4 are in series.
We find their equivalent capacitance from
$$1/C_5 = 1/C_3 + 1/C_4 = 1/(2\ \mu\text{F}) + 1/(5\ \mu\text{F}),\ \text{which gives}\ C_5 = 1.43\ \mu\text{F}.$$
We find the equivalent capacitance of C_2 and C_5,
which are in parallel:
$$C_6 = C_2 + C_5 = 4\ \mu\text{F} + 1.43\ \mu\text{F} = 5.43\ \mu\text{F}.$$
Finally, we find the equivalent capacitance of C_1 and C_6,
which are in series:
$$1/C_{\text{equ}} = 1/C_1 + 1/C_6 = 1/(3\ \mu\text{F}) + 1/(5.43\ \mu\text{F}),$$
which gives $C_{\text{equ}} = \boxed{1.93\ \mu\text{F}}$.

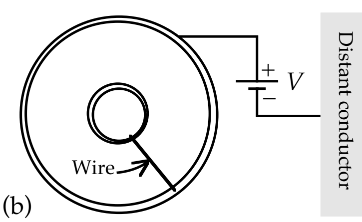


34. When the uncharged plate is placed between the two charged plates, charges will separate so that there is a charge $+Q$ on the side facing the negative plate and a charge $-Q$ on the side facing the positive plate. Thus we have two capacitors in series, with an equivalent capacitance:
$$1/C = 1/C_1 + 1/C_2 = 1/(\varepsilon_0 A/x) + 1/[\varepsilon_0 A/(D - x - d)] = (x + D - x - d)/\varepsilon_0 A,\ \text{which gives}$$
$$C = \boxed{\varepsilon_0 A/(D - d),\ \text{independent of}\ x}.$$
Note that this is a parallel-plate capacitor with separation $D - d$.

35. Because the potential from the outer conductor is constant inside, the potential difference between the two conductors is due to the inner conductor, which is equivalent to a point charge:
$$V = (Q/4\pi\varepsilon_0)(1/r - 1/R),\ \text{so the capacitance is}$$
$$C = Q/V = 4\pi\varepsilon_0 rR/(R - r)$$
$$= 4\pi(8.85 \times 10^{-12}\ \text{F/m})(3 \times 10^{-3}\ \text{m})(12 \times 10^{-3}\ \text{m})/[(12 - 3) \times 10^{-3}\ \text{m}]$$
$$= 4.5 \times 10^{-13}\ \text{F} = \boxed{0.45\ \text{pF}}.$$
When a wire connects the two spheres, they must be at the same potential and all the charge will be on the outer sphere. The potential of the charged outer sphere is
$$V = Q/4\pi\varepsilon_0 R,\ \text{so the capacitance is}$$
$$C = Q/V = 4\pi\varepsilon_0 R$$
$$= 4\pi(8.85 \times 10^{-12}\ \text{F/m})(12 \times 10^{-3}\ \text{m})$$
$$= 1.33 \times 10^{-12}\ \text{F} = \boxed{1.33\ \text{pF}}.$$

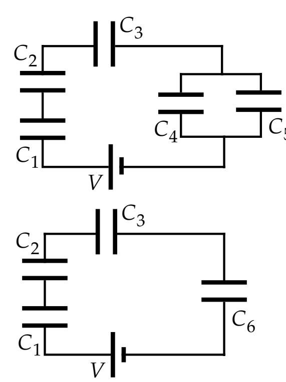


(a)



(b)

36. From the circuit, we see that C_4 and C_5 are in parallel, with an equivalent capacitance
$$C_6 = C_4 + C_5 = 18\ \mu\text{F} + 18\ \mu\text{F} = 36\ \mu\text{F}.$$
We now have four capacitors in series:
$$1/C_{\text{equ}} = 1/C_1 + 1/C_2 + 1/C_3 + 1/C_6$$
$$= 1/(18\ \mu\text{F}) + 1/(18\ \mu\text{F}) + 1/(18\ \mu\text{F}) + 1/(36\ \mu\text{F}),$$
which gives
$$C_{\text{equ}} = \boxed{5.1\ \mu\text{F}}.$$



$\boxed{37}$. (a) From the circuit, we see that C_2 and C_5 are in series and find their equivalent capacitance from

$1/C_6 = 1/C_2 + 1/C_5$

$= 1/(2\ \mu\text{F}) + 1/(5\ \mu\text{F})$, which gives

$C_6 = 1.43\ \mu\text{F}$.

From the new circuit, we see that C_1 and C_6 are in parallel, with an equivalent capacitance

$C_7 = C_1 + C_6 = 1\ \mu\text{F} + 1.43\ \mu\text{F} = 2.43\ \mu\text{F}$.

From the new circuit, we see that C_3 and C_7 are in series and find their equivalent capacitance from

$1/C_{\text{equ}} = 1/C_3 + 1/C_7$

$= 1/(3\ \mu\text{F}) + 1/(2.43\ \mu\text{F})$, which gives

$C_{\text{equ}} = \boxed{1.34\ \mu\text{F}}$.

(b) The charge on the equivalent capacitor is also the charge on C_3 and C_7:

$Q_{\text{equ}} = Q_3 = Q_7 = C_{\text{equ}}V_{ab} = (1.34\ \mu\text{F})(300\ \text{V}) = \boxed{402\ \mu\text{C}}$.

We find the potential difference between c and b from

$V_{cb} = Q_7/C_7 = (402\ \mu\text{C})/(2.43\ \mu\text{F}) = 165\ \text{V}$.

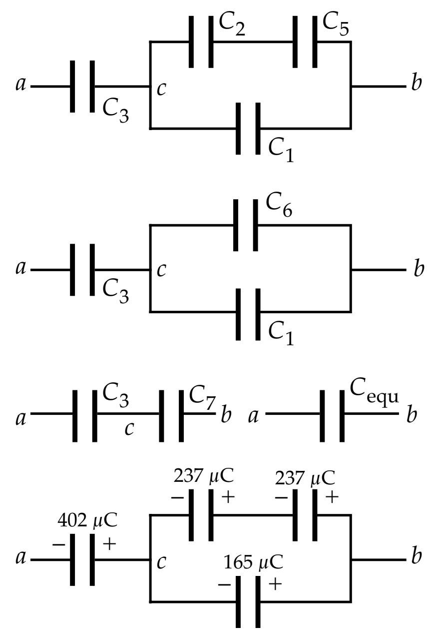
The charge on C_6 is also the charge on C_2 and C_5:

$Q_6 = Q_2 = Q_5 = C_6V_{cb} = (1.43\ \mu\text{F})(165\ \text{V}) = \boxed{237\ \mu\text{C}}$.

The charge on C_1 is

$Q_1 = C_1V_{cb} = (1\ \mu\text{F})(165\ \text{V}) = \boxed{165\ \mu\text{C}}$.

Because point b is at the higher potential, the charges are as shown in the diagram.



38. Although there are no apparent series or parallel combinations in the circuit that can be reduced, we use symmetry to simplify the circuit. The top and bottom paths from a to b are equivalent, so we have

$V_c = V_d$,

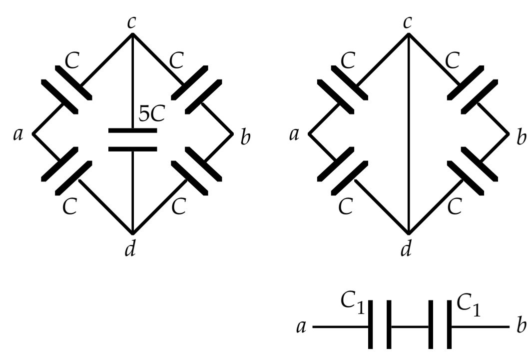
which means there is no potential difference across and no charge on the $5C$ capacitor. The circuit will not change if we replace the $5C$ capacitor with a wire. The left and right sides have two capacitors in parallel, with equivalent capacitance

$C_1 = C + C = 2C$.

We combine these two capacitors in series to find the equivalent capacitance of the circuit:

$1/C_{\text{equ}} = 1/2C + 1/2C$, which gives

$C_{\text{equ}} = \boxed{C}$.



39. To find the equivalent capacitance between a and b, we redraw the circuit, and see that there are two capacitors in series in the right branch:

$$1/C_1 = 1/C + 1/C, \text{ which gives } C_1 = \tfrac{1}{2}C.$$

For the two capacitors in parallel between d and b we have

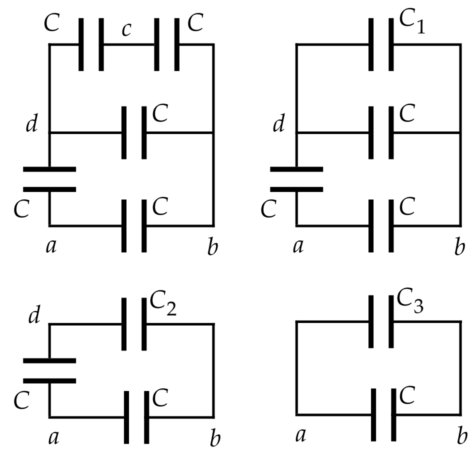
$$C_2 = C_1 + C = \tfrac{1}{2}C + C = \tfrac{3}{2}C.$$

For the two capacitors in series between a and b we have

$$1/C_3 = 1/C + 1/C_2, \text{ which gives } C_3 = 0.6C.$$

For the equivalent capacitance, we have

$$C_{\text{equ},ab} = C + C_3 = C + 0.6C = \boxed{1.6C}.$$



To find the equivalent capacitance between a and c, we redraw the circuit, and use symmetry to simplify the circuit. The top and bottom paths from a to c are equivalent, so we have

$$V_b = V_d,$$

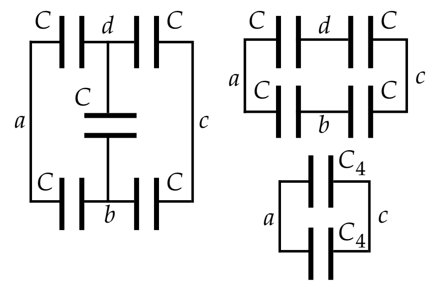
which means there is no potential difference across and no charge on the middle capacitor. The circuit will not change if we remove it.

The top and bottom branches have two capacitors in series:

$$1/C_4 = 1/C + 1/C, \text{ which gives } C_4 = \tfrac{1}{2}C.$$

We combine these two capacitors in parallel to find the equivalent capacitance of the circuit:

$$C_{\text{equ},ac} = C_4 + C_4 = \tfrac{1}{2}C + \tfrac{1}{2}C = \boxed{C}.$$

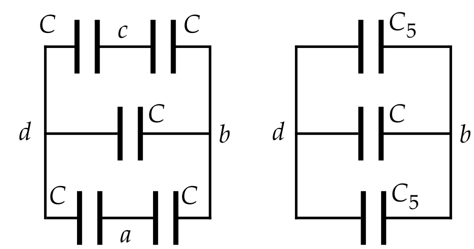


To find the equivalent capacitance between b and d, we redraw the circuit, and see that the top and bottom paths have two capacitors in series:

$$1/C_5 = 1/C + 1/C, \text{ which gives } C_5 = \tfrac{1}{2}C.$$

We now have three capacitors in parallel, with equivalent capacitance

$$C_{\text{equ},bd} = C_5 + C_5 + C = \tfrac{1}{2}C + \tfrac{1}{2}C + C = \boxed{2C}.$$



40. (a) For capacitor C_1, we have

$$U_1 = \tfrac{1}{2}q_1^2/C_1 = \tfrac{1}{2}(4\ \mu\text{C})^2/(175\ \mu\text{F}) = 0.046\ \mu\text{J}.$$

For capacitor C_2, we have

$$U_2 = \tfrac{1}{2}C_2V_2^2 = \tfrac{1}{2}(18\ \mu\text{F})(3\ \text{V})^2 = 81\ \mu\text{J}.$$

The total energy stored in the two capacitors is

$$U = U_1 + U_2 = 0.046\ \mu\text{J} + 81\ \mu\text{J} = \boxed{81\ \mu\text{J}}.$$

(b) When the negatively charged plate of C_1 is connected to the positively charged plate of C_2 we have a single equivalent capacitor, with

$$q_{\text{net}} = |q_2 - q_1| = |(18\ \mu\text{F})(3\ \text{V}) - 4\ \mu\text{C}| = \boxed{50\ \mu\text{C}},$$

$$C_{\text{equ}} = C_1 + C_2 = 175\ \mu\text{F} + 18\ \mu\text{F} = \boxed{193\ \mu\text{F}}, \text{ and}$$

$$V = q_{\text{net}}/C_{\text{equ}} = 50\ \mu\text{C}/193\ \mu\text{F} = \boxed{0.26\ \text{V}}.$$

The total energy stored in the system becomes

$$U = \tfrac{1}{2}q_{\text{net}}^2/C_{\text{equ}} = \tfrac{1}{2}(50\ \mu\text{C})^2/(193\ \mu\text{F})^2 = \boxed{6.5\ \mu\text{J}}.$$

41. (a) We find the capacitance C_2 from

$C_2 = Q_2/V_2 = (25\ \mu\text{C})/(5\ \text{V}) = \boxed{5.0\ \mu\text{F}}$.

(b) Because $Q_1 = Q_2$, the capacitors can be considered to be in series. The charge on the equivalent capacitor is

$Q = Q_1 = Q_2 = \boxed{25\ \mu\text{C}}$.

42. Because the equivalent capacitance for a parallel combination is the sum, we see that there must be some series combination as well. Because the required result of 4.968 μF is greater than 4 μF, we try the 4-μF capacitor in parallel with a series combination of the others. The equivalent capacitance of some of the combinations are

2-μF & 3-μF: $(2\ \mu\text{F})(3\ \mu\text{F})/(2\ \mu\text{F} + 3\ \mu\text{F}) = 1.2\ \mu\text{F}$;
2-μF & 5-μF: $(2\ \mu\text{F})(5\ \mu\text{F})/(2\ \mu\text{F} + 5\ \mu\text{F}) = 1.4\ \mu\text{F}$;
2-μF & 3-μF & 5-μF: $(1.2\ \mu\text{F})(5\ \mu\text{F})/(1.2\ \mu\text{F} + 5\ \mu\text{F}) = 0.97\ \mu\text{F}$.

We get the desired result by putting the 4-μF capacitor in parallel with a series combination of the 2-μF, 3-μF, and 5-μF capacitors.

43. Because the charge is constant, we have

$Q = C_{\text{teflon}}V_{\text{teflon}} = C_{\text{plexiglas}}V_{\text{plexiglas}}$, or $V_{\text{plexiglas}}/V_{\text{teflon}} = C_{\text{teflon}}/C_{\text{plexiglas}} = \kappa_{\text{teflon}}/\kappa_{\text{plexiglas}}$;
$V_{\text{plexiglas}}/(600\ \text{V}) = 2.1/3.4$, which gives $V_{\text{plexiglas}} = \boxed{370\ \text{V}}$.

44. We find the dielectric constant from

$\kappa = C/C_0 = (Q/V)/(Q/V_0) = V_0/V = (4\ \text{V})/(3.6\ \text{V}) = \boxed{1.1\text{V}}$.

45. For the same energy stored at the same potential difference, we have

$U = \frac{1}{2}CV^2 = \frac{1}{2}(\kappa\varepsilon_0 A/d)V^2$;
$4 \times 10^6\ \text{J} = \frac{1}{2}[(3)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)A/(1 \times 10^{-3}\ \text{m})](12\ \text{V})^2$, which gives
$A = \boxed{2.1 \times 10^{12}\text{m}^2}$.

46. (a) Using the results from Problem 17, we have

$C = L2\pi\kappa\varepsilon_0/\ln(b/a) = \kappa C_0 = (2.5)(5.67 \times 10^{-10}\ \text{F}) = \boxed{1.42 \times 10^{-9}\ \text{F}}$.

(b) The energy stored in 10 m of cable is

$U_1 = \frac{1}{2}CV^2 = \frac{1}{2}\kappa C_0 V^2 = \kappa U_0 = (2.5)(2.83 \times 10^{-4}\ \text{J}) = \boxed{7.08 \times 10^{-4}\ \text{J}}$.

Because the capacitance is directly proportional to the length, the energy stored in 1 km of cable is

$U_2 = [(10^3\ \text{m})/(10\ \text{m})]U_1 = \boxed{7.08 \times 10^{-2}\ \text{J}}$.

47. (a) The area of the parallel-pate capacitor is

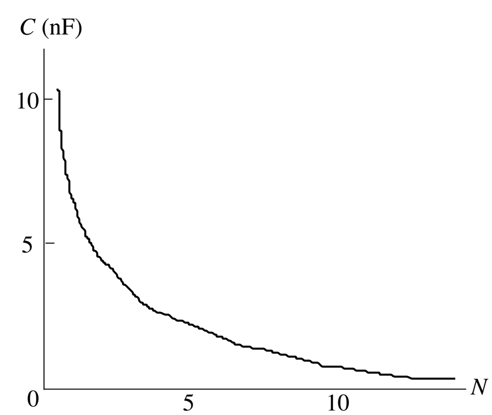
$A = (0.20\ \text{m})(0.15\ \text{m}) = 0.030\ \text{m}^2$, and its plate Separation is $d = 7.7\ \text{mm}/100 = 7.7 \times 10^{-5}\ \text{m}$.
Its capacitance is then

$$C = \kappa\varepsilon_0 A/d$$
$$= (2.9)(8.85 \times 10^{-12}\ \text{F/m})(0.030\ \text{m}^2)/(7.7 \times 10^{-5}\ \text{m})$$
$$= 1.0 \times 10^{-8}\ \text{F} = \boxed{10\ \text{nF}}.$$

(b) Since C is proportional to κ, the actual value of κ is

$\kappa = (4.6\ \text{nF}/\ 10\ \text{nF})(2.9) = \boxed{1.3}$.

(c)



48. With $A = 2\text{ cm}^2 = 2 \times 10^{-4}\text{m}^2$, $d = 0.1\text{ nm} = 1 \times 10^{-10}$ m, and $\kappa = 10$, the capacitance is

$C = \kappa\varepsilon_0 A/d = (10)(8.85 \times 10^{-12}\text{ F/m})(2 \times 10^{-4}\text{m}^2)/(1 \times 10^{-10}\text{ m}) = \boxed{2 \times 10^{-4}\text{ F}}$.

49. The capacitance of an isolated sphere in air is $C_0 = 4\pi\varepsilon_0 R$. When it is embedded in a dielectric, we have

$C = \kappa C_0$, so the change is

$C - C_0 = (\kappa - 1)C_0 = \boxed{(\kappa - 1)4\pi\varepsilon_0 R}$.

The sign of the induced charge on the surface of the dielectric will be opposite to that of the original charge. If $E_0 = \sigma/\varepsilon_0$ is the original electric field just outside the sphere, we have

$E = E_0/\kappa = E_0 + E_{\text{ind}}$;

$\sigma/\kappa\varepsilon_0 = \sigma/\varepsilon_0 + \sigma_{\text{ind}}/\varepsilon_0$, which gives $\sigma_{\text{ind}}/\sigma = \boxed{(\kappa - 1)/\kappa}$.

50. We find the initial separation from

$E_0 = V/d$;

$0.90 \times 10^6\text{ V/m} = (12 \times 10^3\text{ V})/d$, which gives $d = 0.013$ m.

Because the capacitance does not change, we have

$C = \varepsilon_0 A/d = \kappa\varepsilon_0 A/d'$, which gives

$d' = \kappa d = 1.5(0.013\text{ m}) = \boxed{0.020\text{ m}}$.

51. If we write the energies as $U_0 = \frac{1}{2}q_0^2/C_0$ and $U = \frac{1}{2}q^2/C$, the ratio is

$U/U_0 = (q/q_0)^2(C_0/C) = (q/q_0)^2(1/\kappa)$;

$3 = (q/q_0)^2(1/1.8)$, which gives $q = \boxed{2.3q_0}$.

52. (a) For a coaxial cable, we have

$C = L2\pi\kappa\varepsilon_0/\ln(b/a)$

$= (100\text{ m})(2\pi)(2.2)(8.85 \times 10^{-12}\text{ F/m})/\ln[(5.0\text{ mm})/(3.5\text{ mm})] = \boxed{3.4 \times 10^{-8}\text{ F}}$.

(b) The charge on the inner (and the outer) conductor is

$Q = CV$

$= (3.4 \times 10^{-8}\text{ F})(5.0 \times 10^2\text{ V}) = \boxed{1.7 \times 10^{-5}\text{ C}}$.

The energy stored is

$U = \frac{1}{2}CV^2$

$= \frac{1}{2}(3.4 \times 10^{-8}\text{ F})(5.0 \times 10^2\text{ V})^2 = \boxed{4.3 \times 10^{-3}\text{ J}}$.

53. We can consider the system to be two capacitors in series:

$1/C = 1/C_1 + 1/C_2$

$= (D - d)/\varepsilon_0 A + d/\kappa\varepsilon_0 A$

$= (D - d + d/\kappa)]/\varepsilon_0 A$, which gives

$C = \boxed{\kappa\varepsilon_0 A/[d + \kappa(D - d)]}$.

54. From Problem 53, we have

$C = \kappa\varepsilon_0 A/[d + \kappa(D - d)]$.

The charge on the plates is

$Q = CV = \kappa\varepsilon_0 AV/[d + \kappa(D - d)]$, so the electric field in the empty space is

$E_0 = \sigma/\varepsilon_0 = Q/\varepsilon_0 A = \kappa V/[(d + \kappa(D - d)]$

$= (1.8)(600\text{ V})/[0.6 \times 10^{-2}\text{ m} + (1.8)(1.6 \times 10^{-2}\text{ m} - 0.6 \times 10^{-2}\text{ m})] = \boxed{4.5 \times 10^4\text{ V/m}}$.

The electric field in the dielectric is

$E = E_0/\kappa = (4.5 \times 10^4\text{ V/m})/1.8 = \boxed{2.5 \times 10^4\text{ V/m}}$.

$\boxed{55}$. The free charge on the capacitor is

$$Q = CV = (\kappa\varepsilon_0 A/d)V = \kappa\varepsilon_0 AV/d$$
$$= \kappa(8.85 \times 10^{-12}\ \text{F/m})(10 \times 10^{-4}\text{m}^2)(300\ \text{V})/(5 \times 10^{-3}\ \text{m}) = 5.3 \times 10^{-10}\kappa\ \text{C}.$$

For the materials, we have

air, $\kappa = 1$; $Q = \boxed{5.3 \times 10^{-10}\ \text{C}}$;
paper, $\kappa = 3.7$; $Q = \boxed{2.0 \times 10^{-9}\ \text{C}}$;
neoprene, $\kappa = 6.7$; $Q = \boxed{3.6 \times 10^{-9}\ \text{C}}$;
Bakelite, $\kappa = 4.9$; $Q = \boxed{2.6 \times 10^{-9}\ \text{C}}$;
strontium titanate, $\kappa = 332$; $Q = \boxed{1.8 \times 10^{-7}\ \text{C}}$.

56. Using the dielectric strength of plexiglas, we find the separation of the plates:

$$E = V/d;$$
$$d = V_{\max}/E_{\max} = (6 \times 10^3\ \text{V})/(2.8 \times 10^6\ \text{V/m}) = 2.14 \times 10^{-3}\ \text{m}.$$

When the plexiglas is removed, the capacitance is

$$C_0 = \varepsilon_0 A/d$$
$$= (8.85 \times 10^{-12}\ \text{F/m})(0.80 \times 10^{-4}\text{m}^2)/(2.14 \times 10^{-3}\ \text{m}) = 3.3\times 10^{-13}\ \text{F}.$$

The maximum voltage with air between the plates is

$$V_0 = E_{0\max}d$$
$$= (3 \times 10^6\ \text{V/m})(2.14 \times 10^{-3}\ \text{m}) = 6.4 \times 10^3\ \text{V}.$$

The maximum charge the plates can hold now is

$$Q_0 = C_0V_0$$
$$= (3.3 \times 10^{-13}\ \text{C})(6.4 \times 10^3\ \text{V}) = \boxed{2.1 \times 10^{-9}\ \text{C}}.$$

57. Using the result of Problem 35, we know that the capacitance is

$$C = \kappa C_0 = \kappa 4\pi\varepsilon_0 r_1 r_2/(r_2 - r_1).$$

With air between the shells, the energy is

$$U_0 = \tfrac{1}{2}Q^2/C_0.$$

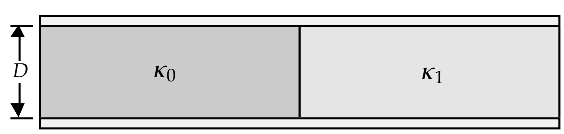
When the dielectric is added, the charge does not change, so the energy is

$$U = \tfrac{1}{2}Q^2/C = \tfrac{1}{2}Q^2/\kappa C_0.$$

The change in energy is

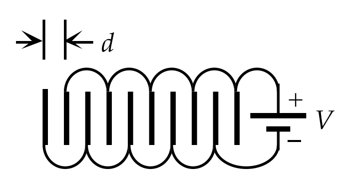
$$U - U_0 = \tfrac{1}{2}(Q^2/C_0)(1/\kappa - 1) = \tfrac{1}{2}(Q^2/\kappa C_0)(1-\kappa) = \boxed{(1-\kappa)Q^2/2C\ \ \text{(a decrease)}}.$$

58. Because $D \ll L$, we can ignore fringing fields. The potential difference must be the same on each half of the space, so we can treat the system as two capacitors in parallel:



$$C = C_1 + C_2 = \kappa_0\varepsilon_0(\tfrac{1}{2}L^2)/d + \kappa_1\varepsilon_0(\tfrac{1}{2}L^2)/d$$
$$= (\varepsilon_0\tfrac{1}{2}L^2/d)(\kappa_0 + \kappa_1) = \boxed{\tfrac{1}{2}(\kappa_0 + \kappa_1)(\varepsilon_0 L^2/d)}.$$

59. From the diagram, we see that the arrangement is equivalent to 9 capacitors in parallel:



$$C = 9C_1 = 9(\varepsilon_0 A/d)$$
$$= 9(8.85 \times 10^{-12}\ \text{F/m})(6.0 \times 10^{-2}\ \text{m})(8.0 \times 10^{-2}\ \text{m})/(1.2 \times 10^{-3}\ \text{m})$$
$$= 3.2 \times 10^{-10}\ \text{F} = \boxed{0.32\ \text{nF}}.$$

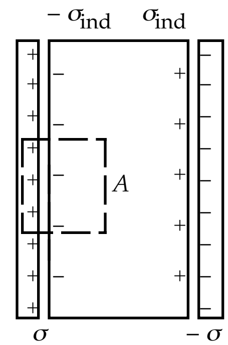
If the region is filled with a dielectric, we have

$$C' = \kappa C = 2.8(0.32\ \text{nF}) = \boxed{0.90\ \text{nF}}.$$

60. We find the induced charge from

$$Q_{\text{ind}} = Q(1 - 1/\kappa) = (18\ \mu\text{C})(1 - 1/4.5) = \boxed{14\ \mu\text{C}}.$$

61. We choose a cylinder with one end inside the conducting plate and the other end in the dielectric for a Gaussian surface. Because there is no field inside the plate and the field is parallel to the sides, the only part of the cylinder with flux through it is the end in the dielectric:



$$\oiint \vec{E}\cdot d\vec{A} = \iint E\,dA = Q_{encl}/\varepsilon_0;$$
$$EA = (\sigma - \sigma_{ind})A/\varepsilon_0, \text{ which gives}$$
$$E = E_0 - E_{ind} = \sigma/\varepsilon_0 - \sigma_{ind}/\varepsilon_0.$$

Because $E_0 = \sigma/\varepsilon_0$, we have
$$E_{ind} = \sigma_{ind}/\varepsilon_0.$$

62. We find the induced surface charge density from
$$\sigma_{ind} = \sigma - \sigma/\kappa = \sigma(1 - 1/\kappa) = (\kappa - 1)(Q/L^2\kappa).$$

The induced surface charge is
$$Q_{ind} = \sigma_{ind}L^2 = (\kappa - 1)(Q/\kappa)$$
$$= (3.5 - 1)(0.3\times10^{-6}\ \text{C})/3.5 = \boxed{2.1\times10^{-7}\ \text{C}}.$$

The field in the dielectric is
$$E = E_0/\kappa = \sigma/\kappa\varepsilon_0 = Q/L^2\kappa\varepsilon_0$$
$$= (0.3\times10^{-6}\ \text{C})/[(0.22\ \text{m})^2(3.5)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)] = \boxed{2.2\times10^5\ \text{V/m}}.$$

The energy stored in the capacitor is
$$U = \tfrac{1}{2}Q^2/C = \tfrac{1}{2}Q^2d/\kappa\varepsilon_0L^2$$
$$= \tfrac{1}{2}(0.3\times10^{-6}\ \text{C})^2(1.8\times10^{-3}\ \text{m})/[(3.5)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.22\ \text{m})^2] = \boxed{5.4\times10^{-5}\ \text{J}}.$$

63. For a polar dielectric we have
$$\kappa = 1 + a/T, \text{ so}$$
$$C = \kappa\varepsilon_0A/d = \kappa C_0 = (1 + a/T)C_0.$$

With the given data we have
$$3.2\ \mu\text{F} = (1 + a/296\ \text{K})C_0 \quad\text{and}\quad 2.65\ \mu\text{F} = (1 + a/360\ \text{K})C_0.$$

When we solve these two equations, we get
$$a = 8640\ \text{K} \text{ and } C_0 = 0.106\ \mu\text{F}.$$

At a temperature of 48°C, we have
$$C = (1 + 8640\ \text{K}/321\ \text{K})(0.106\ \mu\text{F}) = \boxed{2.96\ \mu\text{F}}.$$

64. $\boxed{\text{The uncharged plate will be sucked in}}$. The charges induced on the surfaces of the inserted plate will be opposite to those on the charged plates. If we consider one of the charged plates, the induced charge of the opposite sign will be closer than the induced charge of the same sign and thus the force of attraction will be greater than the force of repulsion. The same will be true for the other charged plate.

65. (a) Since the two capacitors are in series they must have the same charge:
$$q = C_1V_1 = C_2V_2.$$

Also, the sum of their voltages is V:
$$V = V_1 + V_2.$$

Combine these two equations to obtain
$$V_1 = \boxed{C_2V/(C_1 + C_2)}, \quad V_2 = \boxed{C_1V/(C_1 + C_2)}.$$

(b) Since
$$V_1 = C_2V/(C_1 + C_2) = (3\ \text{nF})V/(2\ \text{nF} + 3\ \text{nF}) = 3V/5 < V_{1max} = 10\ \text{V}, \text{ we have } V < 17\ \text{V}. \text{ Also,}$$
$$V_2 = C_1V/(C_1 + C_2) = (2\ \text{nF})V/(2\ \text{nF} + 3\ \text{nF}) = 2V/5 < V_{2max} = 30\ \text{V}, \text{ we have } V < 75\ \text{V}.$$

To satisfy both inequalities we must have $V < 17$ V. So $\boxed{V_{max} = 17\ \text{V}}$.

66. (a) Two capacitors connected in parallel must have the same potential difference, so
$V_2 = \boxed{V}$.
(b) The single equivalent capacitor is still subject to the same voltage difference, $\boxed{V}$.
(c) Since the voltage difference across the two capacitors is the same, and the maximum voltage difference across C_1 is 10 V, less than that across C_2, the maximum voltage difference across the combination is
$\boxed{V_{max} = V_{1max} = 10\text{ V}}$.

67. We take a radius R of 0.5 m for the sphere and a spark distance d of 0.5 cm. Because the spark distance is small, we assume the breakdown field is constant, so the required potential is $V = Ed$. From the potential of a sphere, we have
$V = (1/4\pi\varepsilon_0)(Q/R) = E_{max}d$;
$(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)Q/(0.5\text{ m}) = (3 \times 10^6\text{ V/m})(0.5 \times 10^{-2}\text{ m})$, which gives $\boxed{Q \simeq 10^{-6}\text{ C}}$.

68. (a) Because a single potential is available, from $Q = CV$ we see that the maximum charge will be produced by the maximum capacitance. For a parallel-plate capacitor, $C = \kappa\varepsilon_0 A/d$. We need a system with maximum area and minimum separation. The minimum separation is 5 mm, and the maximum area possible is 150 cm^2. (Note that if we make a number of smaller capacitors, they will be connected in parallel to produce the maximum capacitance. This is the same as a single capacitor.) The system consists of 2 aluminum plates of area 150 cm^2, separated by 5 mm, with a 150 cm^2 piece of Bakelite between the plates. The designed capacitance is
$C = \kappa\varepsilon_0 A/d$
$= (4.9)(8.85 \times 10^{-12}\text{ F/m})(150 \times 10^{-4}\text{m}^2)/(5 \times 10^{-3}\text{ m}) = 1.30 \times 10^{-10}\text{ F}$.
The charge on the plates is
$Q = CV = (1.30 \times 10^{-10}\text{ F})(1200\text{ V}) = \boxed{1.56 \times 10^{-7}\text{ C}}$.
The energy stored is
$U = \frac{1}{2}CV^2 = \frac{1}{2}(1.30 \times 10^{-10}\text{ F})(1200\text{ V})^2 = \boxed{9.37 \times 10^{-5}\text{ J}}$.
(b) Because a single potential is available, from $E = V/d$ we see that the maximum field will be produced by the minimum separation. The Bakelite is not needed to have this electric field, so the system is the same, but with no Bakelite. The electric field is
$E_0 = (1200\text{ V})/(5 \times 10^{-3}\text{ m}) = \boxed{2.4 \times 10^5\text{ V/m}}$.

69. (a) We find the equivalent capacitance for N capacitors in series from
$1/C_{series} = \sum(1/C_i) = N/C_1$, which gives $C_{series} = C_1/N$.
The energy stored is
$U_{series} = \frac{1}{2}C_{series}V^2 = \boxed{\frac{1}{2}C_1V^2/N}$.
(b) We find the equivalent capacitance for N capacitors in parallel from
$C_{parallel} = \sum C_i = NC_1$.
The energy stored is
$U_{parallel} = \frac{1}{2}C_{parallel}V^2 = \boxed{\frac{1}{2}NC_1V^2}$.
(c) The equivalent capacitance does not change, so we have
$U_{series} = \frac{1}{2}C_{series}V^2 = \frac{1}{2}Q^2/C_{series} = \boxed{\frac{1}{2}Q^2N/C_1}$;
$U_{parallel} = \frac{1}{2}Q^2/C_{parallel} = \boxed{\frac{1}{2}Q^2/NC_1}$.

70. (a) Because there is no electric field within the metal plate, the system is two capacitors in series. If we call the separation for one of them $d_1 = 1d/3$, we find the equivalent capacitance from

$1/C_{\text{metal}} = 1/C_1 + 1/C_1 = d_1/\varepsilon_0 A + d_1/\varepsilon_0 A$, which gives

$C_{\text{metal}} = \varepsilon_0 A/2d_1 = 3\varepsilon_0 A/2d$

$= 3(8.85 \times 10^{-12}\ \text{F/m})(0.28\ \text{m}^2)/2(1.5\times 10^{-2}\ \text{m}) = 2.5 \times 10^{-10}\ \text{F} = \boxed{0.25\ \text{nF}}$.

(b) Because there is no field within the metal, the surface charge density induced on the intermediate plate is

$\sigma_{\text{ind}} = Q/A = Q/(0.28\ \text{m}^2) = 3.57Q\ \text{C/m}^2$.

(c) The original energy is

$U_1 = \frac{1}{2}Q^2/C_0 = Q^2 d/2\varepsilon_0 A$.

The new energy is

$U_2 = \frac{1}{2}Q^2/C_{\text{metal}} = Q^2 d/3\varepsilon_0 A$.

The ratio is

$\boxed{U_2/U_1 = 2/3\ \ \text{(a decrease)}}$.

(d) When a dielectric is inserted, the system is three capacitors in series.
We find the equivalent capacitance from

$1/C_{\text{dielectric}} = 1/C_1 + 1/C_2 + 1/C_3$

$= d_1/\varepsilon_0 A + d_1/\kappa\varepsilon_0 A + d_1/\varepsilon_0 A$

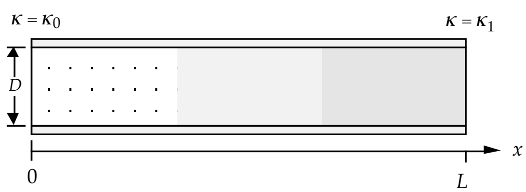
$= (2d_1 + d_1/\kappa)/\varepsilon_0 A = (2 + 1/\kappa)\, d_1/\varepsilon_0 A$, which gives

$C_{\text{dielectric}} = (3\varepsilon_0 A/d)[\kappa/(2\kappa + 1)]$.

The ratio is

$\boxed{C_{\text{dielectric}}/C_{\text{metal}} = 2\kappa/(2\kappa + 1)}$.

71.



We find the capacitance of the strip of the dielectric at x, with width dx, from

$\mathrm{d}C = \kappa\varepsilon_0\, \mathrm{d}A/D = \kappa\varepsilon_0 L\, \mathrm{d}x/D$.

The strips that make up the capacitor are in parallel, so the equivalent capacitance is

$$C = \int \mathrm{d}C = \int_0^L \frac{\kappa\varepsilon_0 L}{D}\,\mathrm{d}x = \varepsilon_0 \int_0^L \left[\kappa_0 + \frac{(\kappa_1 - \kappa_0)x}{L}\right]\frac{L}{D}\,\mathrm{d}x$$

$$= \frac{\varepsilon_0 L}{D}\left[\kappa_0 x + \frac{(\kappa_1 - \kappa_0)x^2}{2L}\right]\Bigg|_0^L = \frac{\varepsilon_0 L}{D}\left[\kappa_0 L + \frac{(\kappa_1 - \kappa_0)L^2}{2L}\right], \text{which reduces to}$$

$C = \boxed{\frac{1}{2}(\kappa_0 + \kappa_1)(\varepsilon_0 L^2/d)}$.

72. Using the estimates given, we have

$C = Q/V$

$\simeq (10^2\ \text{C})/(10^8\ \text{V}) \simeq \boxed{10^{-6}\ \text{F}}$.

The energy stored in the capacitor is

$U = \frac{1}{2}CV^2 = \frac{1}{2}QV$

$\simeq \frac{1}{2}(10^2\ \text{C})(10^8\ \text{V}) \simeq \boxed{10^{10}\ \text{J}}$.

73. We find the equivalent capacitance of the circuit.

B and D are in parallel:

$$C_1 = C_B + C_D = 4.3\ \mu\text{F} + 2.1\ \mu\text{F} = 6.4\ \mu\text{F}.$$

We now have three capacitors in series:

$$1/C_{\text{equ}} = 1/C_A + 1/C_1 + 1/C_C$$
$$= 1/(5.4\ \mu\text{F}) + 1/(6.4\ \mu\text{F}) + 1/(3.2\ \mu\text{F}),$$

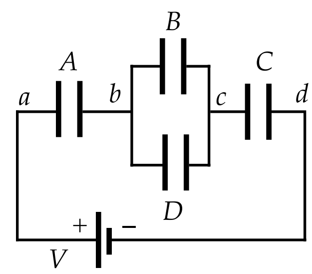
which gives $C_{\text{equ}} = 1.53\ \mu\text{F}$.

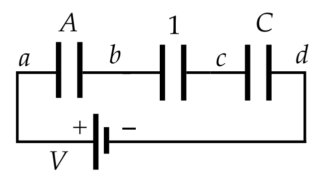
We find the charge on the equivalent capacitor, which is also the charge on each capacitor in series, from

$$Q_{\text{equ}} = Q_A = Q_1 = Q_C = C_{\text{equ}} V_{ab}$$
$$= (1.53\ \mu\text{F})(3000\ \text{V}) = 4.6 \times 10^3\ \mu\text{C}.$$

We find the potential differences from

$$V_A = V_{ac} = Q_A/C_A = (4.6 \times 10^3\ \mu\text{C})/(5.4\ \mu\text{F}) = \boxed{8.5 \times 10^2\ \text{V}};$$
$$V_B = V_D = V_{cd} = Q_1/C_1 = (4.6 \times 10^3\ \mu\text{C})/(6.4\ \mu\text{F}) = \boxed{7.2 \times 10^2\ \text{V}};$$
$$V_C = V_{db} = Q_C/C_C = (4.6 \times 10^3\ \mu\text{C})/(3.2\ \mu\text{F}) = \boxed{1.43 \times 10^3\ \text{V}}.$$





74. (a) We find the capacitance from

$$C_0 = \varepsilon_0 A/d$$
$$= (8.85 \times 10^{-12}\ \text{F/m})(0.40\ \text{m}^2)/(3.0 \times 10^{-3}\ \text{m}) = \boxed{1.2 \times 10^{-9}\ \text{F}}.$$

The maximum voltage is

$V_{\max} = E_{\max} d$, so the maximum charge is

$$Q_{\max} = C_0 V_{\max} = C_0 E_{\max} d = (1.2 \times 10^{-9}\ \text{F})(2.7 \times 10^6\ \text{V/m})(3.0 \times 10^{-3}\ \text{m}) = \boxed{9.7 \times 10^{-6}\ \text{C}}.$$

(b) $E_{\max} = Q_{\max}/Cd = Q_{\max}/\kappa C_0 d$

$$= (9.7 \times 10^{-6}\ \text{C})/(6.0)(1.2 \times 10^{-9}\ \text{F})(3.0 \times 10^{-3}\ \text{m}) = \boxed{4.5 \times 10^5\ \text{V/m}}.$$

75. The energy stored in the capacitor is

$$U_0 = \tfrac{1}{2} C_0 V_0^2 = \tfrac{1}{2}(3.0 \times 10^{-6}\ \text{F})(1500\ \text{V})^2 = \boxed{3.4\ \text{J}}.$$

Because the source is disconnected, the charge on the capacitor does not change, and we have

$$C = \kappa C_0;\ V = V_0/\kappa.$$

The energy stored after the dielectric is inserted is

$$U = \tfrac{1}{2} C V^2 = \tfrac{1}{2}\kappa C_0 (V_0/\kappa)^2 = (1/\kappa)(\tfrac{1}{2} C_0 V_0^2) = (1/\kappa) U_0.$$

We find the work required to insert the dielectric from

$$W = \Delta U = (1/\kappa - 1) U_0$$
$$= (1/2.8 - 1)(3.4\ \text{J}) = \boxed{-2.2\ \text{J}}.$$

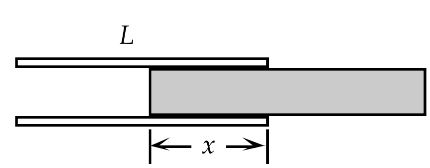
The negative value means that the dielectric is drawn into the region between the plates.

76. We call the length of a plate L, so that $A = L^2$. We can treat the system as two capacitors in parallel:

$$C = C_{\text{dielectric}} + C_{\text{air}}$$
$$= \kappa\varepsilon_0 L x/d + \varepsilon_0 L(L - x)/d$$
$$= (\varepsilon_0 L^2/d)[\kappa(x/L) + 1 - x/L)]$$
$$= \boxed{(\varepsilon_0 A/d)[1 + (\kappa - 1)x/A^{1/2}]}.$$

The energy stored is

$$U = \tfrac{1}{2} C V^2 = \boxed{(\varepsilon_0 A V^2/2d)[1 + (\kappa - 1)x/A^{1/2}]}.$$

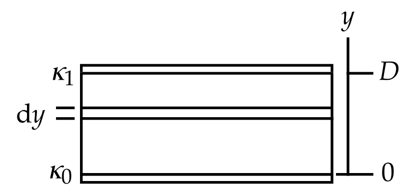


77. We take a strip of the dielectric perpendicular to the y-axis, with thickness Δy, as a capacitor. The capacitance of this strip is

$C_y = \kappa\varepsilon_0 L^2/\Delta y.$

All of the strips from $y = 0$ to $y = D$ are in series, so we find the total capacitance from

$1/C = \sum(1/C_i) = \sum(\Delta y/\kappa\varepsilon_0 A).$



In the limit $\Delta y \to 0$, this sum becomes an integral:

$$\frac{1}{C} = \int_0^D \frac{\mathrm{d}y}{\kappa\varepsilon_0 L^2} = \frac{1}{\varepsilon_0 L^2}\int_0^D \frac{\mathrm{d}y}{\kappa_0 + \left[\left(\kappa_1 - \kappa_0\right)y/D\right]}$$

$$= \frac{D}{\varepsilon_0 L^2\left(\kappa_1 - \kappa_0\right)} \ln\left\{\kappa_0 + \left[\left(\kappa_1 - \kappa_0\right)y/D\right]\right\}\Big|_0^D = \frac{D}{\varepsilon_0 L^2\left(\kappa_1 - \kappa_0\right)} \ln\left[\frac{\kappa_0 + \left(\kappa_1 - \kappa_0\right)}{\kappa_0}\right]$$

$$= \frac{D}{\varepsilon_0 L^2\left(\kappa_1 - \kappa_0\right)} \ln\left(\frac{\kappa_1}{\kappa_0}\right).$$

The capacitance is

$C = \boxed{(\kappa_1 - \kappa_0)\varepsilon_0 L^2/[D\ln(\kappa_1/\kappa_0)]}.$

78. For two capacitors in series, the equivalent capacitance is

$C_{\text{equ, series}} = C_1 C_2/(C_1 + C_2).$

If we subtract this from one of the capacitances, we have

$C_i - C_{\text{equ, series}} = C_i^2/(C_1 + C_2) > 0.$

Because we can combine a series arrangement successively as pairs, the equivalent capacitance for a series combination is always less than any single capacitance. (Also see Problem 79.)
For capacitors in parallel, the equivalent capacitance is

$C_{\text{equ, parallel}} = \sum C_i = 2\mu\text{F} + 4\mu\text{F} + 9\mu\text{F} = \boxed{15\mu\text{F}}.$

so the equivalent capacitance is always greater than any single capacitance.
Thus, we arrange the three capacitors in series for the smallest equivalent capacitance:

$1/C_{\min} = 1/C_1 + 1/C_2 + 1/C_3 = 1/(2\mu\text{F}) + 1/(4\ \mu\text{F}) + 1/(9\ \mu\text{F})$, which gives $C_{\min} = \boxed{1.2\ \mu\text{F}}.$

79. We find the equivalent capacitance for a series arrangement from

$$\frac{1}{C_{\text{equ}}} = \sum_i \left(\frac{1}{C_i}\right).$$

If we multiply by the value of the jth capacitance, we get

$$\frac{C_j}{C_{\text{equ}}} = \sum_i \left(\frac{C_j}{C_i}\right) = 1 + \sum_{i \neq j}\left(\frac{C_j}{C_i}\right).$$

Because the summation is positive, we have

$C_j/C_{\text{equ}} > 1$, for any value of j.

Thus the equivalent capacitance is less than any of the individual capacitances.

80. To distinguish the two capacitors, we label the one in which the dielectric is inserted A and the other one B. The two identical capacitors in series will be system 1, and the system with the dielectric will be system 2.

For system 1:

We find the equivalent capacitance from

$$C_1 = C_{A1}C_B/(C_{A1} + C_B) = CC/(C + C) = \tfrac{1}{2}C.$$

The charge on the equivalent capacitance is the charge on either capacitor:

$$Q_1 = Q_{A1} = Q_{B1} = C_1V = \tfrac{1}{2}CV.$$

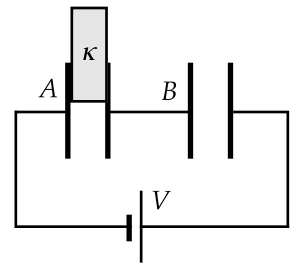
The voltage drops are

$$V_{A1} = Q_{A1}/C_{A1} = (\tfrac{1}{2}CV)/C = \tfrac{1}{2}V;$$

$$V_{B1} = Q_{B1}/C_{B1} = (\tfrac{1}{2}CV)/C = \tfrac{1}{2}V.$$

The stored energy is

$$U_1 = \tfrac{1}{2}C_1V^2 = \tfrac{1}{4}CV^2.$$



For system 2:

We find the equivalent capacitance from

$$C_2 = C_{A2}C_B/(C_{A2} + C_B) = \kappa CC/(\kappa C + C) = [\kappa/(\kappa + 1)]C.$$

The charge on the equivalent capacitance is the charge on either capacitor:

$$Q_2 = Q_{A2} = Q_{B2} = C_2V = [\kappa/(\kappa + 1)]CV.$$

The voltage drops are

$$V_{A2} = Q_{A2}/C_{A2} = [\kappa/(\kappa + 1)]CV/\kappa C = [1/(\kappa + 1)]V;$$

$$V_{B2} = Q_{B2}/C_{B2} = [\kappa/(\kappa + 1)]CV/C = [\kappa/(\kappa + 1)]V.$$

The stored energy is

$$U_2 = \tfrac{1}{2}C_2V^2 = \tfrac{1}{2}[\kappa/(\kappa + 1)]CV^2.$$

The changes are

$$\Delta U = \tfrac{1}{2}[\kappa/(\kappa + 1) - \tfrac{1}{2}]\,CV^2 = \boxed{[(\kappa - 1)/2(\kappa + 1)]\tfrac{1}{2}CV^2};$$

$$\Delta Q_A = \Delta Q_B = [\kappa/(\kappa + 1) - \tfrac{1}{2}]\,CV = \boxed{[(\kappa - 1)/2(\kappa + 1)]CV};$$

$$\Delta V_A = [1/(\kappa + 1) - \tfrac{1}{2}]V = \boxed{[(1 - \kappa)/2(\kappa + 1)]V};$$

$$\Delta V_B = [\kappa/(\kappa + 1) - \tfrac{1}{2}]V = \boxed{[(\kappa - 1)/2(\kappa + 1)]V}.$$

Because $\kappa > 1$, the energy has increased. This energy is supplied by the source as the additional charge moves onto the plates.